

Thermodynamics of large AdS black holes

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ABSTRACT: We consider leading order quantum corrections to the geometry of large AdS black holes in a spherical reduction of four-dimensional Einstein gravity with negative cosmological constant. The Hawking temperature grows without bound with increasing black hole mass, yet the semiclassical back-reaction on the geometry is relatively mild, indicating that observers in free fall outside a large AdS black hole never see thermal radiation at the Hawking temperature. The positive specific heat of large AdS black holes is a statement about the dual gauge theory rather than an observable property on the gravity side. Implications for string thermodynamics with an AdS infrared regulator are briefly discussed.

KEYWORDS: AdS-CFT Correspondence, Black Holes in String Theory.

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1. Introduction

Black holes play an important role in the AdS/CFT correspondence [1–3]. In particular, the so-called large AdS-Schwarzschild black holes, whose horizon area is large in units of the characteristic AdS length scale, correspond to high-temperature thermal states in the dual gauge theory [4, 5]. Another motivation for studying AdS black holes comes from string thermodynamics. The infrared instability of a self-gravitating gas of strings can be regulated by embedding the system in an asymptotically AdS background and in this context large AdS-Schwarzschild black holes are the dominant configurations at sufficiently high total energy [6].

Large AdS-Schwarzschild black holes differ from ordinary Schwarzschild black holes in asymptotically flat spacetime in a number of ways, but perhaps the most striking difference is that the Hawking temperature of large AdS-Schwarzschild black holes grows as a function of the black hole mass [5]. In this sense these black holes have positive specific heat. Due to the curvature in the asymptotic AdS region, Hawking radiation from a black hole gets reflected back and at the quantum level one expects to find equilibrium configurations where a black hole emits Hawking radiation and absorbs particles from the environment at the same rate. In section 3 we consider semiclassical corrections to the AdS-Schwarzschild black hole geometry, which are obtained via spherical reduction of four-dimensional gravity. We look for static solutions to describe equilibrium configurations involving large AdS-Schwarzschild black holes and study some of their physical properties.

Formally there is no upper limit on the mass of a large AdS-Schwarzschild black hole and the Hawking temperature can be arbitrarily high. Does this mean that a large semi-classical black hole is in equilibrium with a high-temperature environment? Our answer is no, but before we get to that let us consider the question more carefully. First of all, in an asymptotically AdS geometry there is infinite gravitational redshift as one goes to spatial infinity and therefore the local temperature goes to zero asymptotically far from a AdS-Schwarzschild black hole, regardless of how high the Hawking temperature is. The question is rather whether there is a region near the black hole, i.e. within a proper distance of order the AdS length scale or so of the event horizon, where the local temperature becomes of order the Hawking temperature.

The answer to this question depends on what kind of observer we have in mind. Let us first consider a fiducial observer, who is kept at rest with respect to the static AdS-Schwarzschild coordinates. Such an observer will indeed measure high temperatures near a large AdS-Schwarzschild black hole but this can be attributed to the acceleration required to keep the observer from falling into the black hole and is the same effect as found outside an ordinary Schwarzschild black hole [7, 8]. In particular, this *fiducial* temperature goes to infinity as we approach the black hole event horizon but we do not expect this to lead to strong back reaction effects for AdS-Schwarzschild black holes anymore than it does for Schwarzschild black holes.

It is more interesting to ask what temperature would be recorded by an observer in free fall near the black hole. This is the relevant temperature when we consider quantum corrections to the black hole geometry. If the black hole is in equilibrium with a layer of high-temperature radiation, then the proper energy density is high in that layer and one would expect large corrections to the geometry. This would have to be reconciled with the fact that in the limit of large black hole mass the spacetime curvature in the horizon region does not grow beyond the AdS scale, as we will see in section 2.1, and thus *a priori* the size of quantum corrections is not expected to grow with increasing black hole mass.

The question of the local temperature outside a black hole also comes up in the context of string thermodynamics in an asymptotically AdS background. The Jeans instability undermines efforts to define a thermodynamic limit in a gravitational theory such as string theory but the troublesome infrared behavior can be regulated by introducing a negative cosmological constant. At weak coupling and low energies there exist spherically symmetric equilibrium configurations consisting of a gas of strings confined to a local region in an asymptotically AdS spacetime. The infrared regulated configurations are not spatially homogeneous and therefore they do not describe a uniform thermal equilibrium state. They do, however, allow the notion of local thermal equilibrium if the magnitude of the cosmological constant is sufficiently small.

Since only massless modes of the string are excited at low energies the string gas consists to a very good approximation of massless radiation. Classical solutions of Einstein gravity with a negative cosmological constant that describe spherically symmetric 'stars' made of self-gravitating radiation were found in [9, 10] and are briefly described in section 2.2. Analogous solutions can be found of the low-energy effective field theory in string theory. The Jeans instability resurfaces as a gravitational instability of the AdS

star configurations at energies of order the AdS scale and at high energies the dominant configurations are large AdS-Schwarzschild black holes [6].

At the semi-classical level, one still expects to find static solutions that describe stars made from self-gravitating radiation. It may be difficult to write them down in closed form but their physical properties will be very similar to those of the corresponding classical solutions. One also expects to find static solutions that describe large black holes in equilibrium with Hawking radiation reflected from the asymptotic AdS region and we look for such solutions in section 3. The transition from AdS stars to large black holes takes place at the AdS scale. Given that we want to have a description in terms of (semi-)classical geometry we have to take the AdS length large compared to the fundamental string length and this means that the AdS scale transition temperature is far below the Hagedorn temperature.¹ If there is a region outside a large AdS black hole that is characterized by a proper temperature of order the Hawking temperature we would have a problem. This is because the minimum Hawking temperature of an AdS black hole is of order the AdS scale and in the limit of very large black holes the Hawking temperature becomes arbitrarily high, higher than the Hagedorn temperature even. Such high temperatures would precipitate black hole nucleation outside the horizon of the large black hole leading to an instability.

We will argue that the above problems are avoided and that an observer in free fall outside a large AdS black hole does not see temperatures anywhere near the Hawking temperature. In fact, to such an observer, a very large AdS-Schwarzschild black hole is an extremely cold object surrounded by almost empty space. We draw this conclusion from an order-of-magnitude analysis of the semiclassical corrections to the classical black hole geometry. The corrections to the total energy of the system include a contribution from the radiation bath surrounding the black hole and based on the size of these corrections we conclude that the energy density of the radiation bath goes to zero in the limit of infinite black hole mass.

Our indirect calculation indicates that large AdS-Schwarzschild black holes have more in common with Schwarzschild black holes in asymptotically flat spacetime than the enormous difference in Hawking temperatures would suggest. From the point of view of semi-classical gravity a large AdS-Schwarzschild black hole appears to be a cold object that emits very little Hawking radiation, just like we know to be the case for an ordinary Schwarzschild black hole of the same mass in a background with zero cosmological constant. The very high Hawking temperature of a large AdS-Schwarzschild black hole characterizes a dual thermal state in a gauge theory via the AdS/CFT correspondence but on the gravity side this temperature is not realized by observers in free fall anywhere in the spacetime geometry outside a large black hole. The statement that large AdS-Schwarzschild black holes have positive specific heat is thus primarily a statement about the dual gauge theory.

¹This is unfortunate in the sense that it prevents us from directly accessing the expected long-string phase near the Hagedorn energy density via AdS regulated equilibrium configurations. On the other hand, the AdS/CFT correspondence provides an indirect view of the high-energy phase via the dual description of large black holes in terms of a gauge theory in a deconfined phase.

2. Classical solutions

In this section we present the classical solutions that will enter into our subsequent discussion. We take spacetime to be four-dimensional. This is mostly to keep the notation simple but it will also be convenient later on when we consider semiclassical corrections since most of the existing literature on spherical reduction deals with four-dimensional theories of gravity. It is straightforward to generalize the classical solutions to higher dimensions.

2.1 AdS-Schwarzschild black holes

The AdS-Schwarzschild metric in four dimensions reads

$$ds^2 = -f_0(r)dt^2 + \frac{dr^2}{f_0(r)} + r^2 d\Omega_2^2, \quad (2.1)$$

where the radial function $f_0(r)$ is

$$f_0(r) = \frac{r^2}{\ell^2} + 1 - \frac{\mu}{r}. \quad (2.2)$$

For $\mu = 0$ the metric reduces to that of four-dimensional anti-de Sitter spacetime with radius of curvature ℓ and cosmological constant $\Lambda = -3\ell^{-2}$. For $\mu > 0$ the metric describes an eternal black hole with an event horizon at $r = r_s$ where r_s is the single real valued zero of $f_0(r)$. The parameter μ is proportional to the mass of the black hole and can be expressed in terms of the horizon radius r_s as follows:

$$\mu = r_s + \frac{r_s^3}{\ell^2}. \quad (2.3)$$

The Hawking temperature for an AdS_4 -Schwarzschild black hole can be obtained by rotating to Euclidean signature in (2.1) and finding the period of Euclidean time that avoids a conical singularity at $r = r_s$. It is given by

$$T_H = \frac{1}{4\pi} f'_0(r_s) = \frac{1}{4\pi} \left(\frac{1}{r_s} + \frac{3r_s}{\ell^2} \right). \quad (2.4)$$

There is a minimum Hawking temperature which is of order the characteristic energy scale of the AdS background, $T \geq \sqrt{3}/2\pi\ell$. Each temperature value above the minimum one is realized for two different values of r_s so there are two branches of AdS-Schwarzschild black holes: 'large' black holes with $r_s > \ell/\sqrt{3}$, and 'small' black holes with $r_s < \ell/\sqrt{3}$.

Let us consider the length scales that enter into our discussion. First of all there is the AdS length ℓ which we take to be large compared to any fundamental length scale such as the string length, i.e. $\ell \gg \ell_s$. This is in order to justify using classical geometry in the first place and to keep quantum corrections under control in our semiclassical considerations later on. It is also useful to have in mind an intermediate length scale ℓ_o that we associate with macroscopic observers who can move around in spacetime, experience tidal effects, measure a local temperature, *etc.* We will be interested in static semi-classical solutions that describe large black holes in equilibrium with an 'atmosphere' consisting of Hawking

radiation or low-energy solutions that describe a non-singular 'star' made of self-gravitating radiation. In either case, the geometry varies on length scales of order the AdS length ℓ or larger and conditions for local thermal equilibrium are satisfied at the ℓ_o scale provided

$$\ell_s \ll \ell_o \ll \ell. \quad (2.5)$$

We will mostly be concerned with large AdS-Schwarzschild black holes and among those it is interesting to consider the very large limit where

$$r_s \approx (\mu \ell^2)^{1/3} \gg \ell. \quad (2.6)$$

Some features are universal in this limit. Consider for example the scalar invariant

$$R_{abcd}R^{abcd} = 12 \left(\frac{2}{\ell^4} + \frac{\mu^2}{r^6} \right), \quad (2.7)$$

where R_{abcd} are components of the Riemann tensor and the right hand side is the value obtained for the AdS-Schwarzschild metric (2.1). Now let $r = r_s$ and take the limit of a very large black hole to obtain

$$R_{abcd}R^{abcd} \Big|_{\text{horizon}} = \frac{36}{\ell^4} \left(1 + O(\ell/\mu)^{2/3} \right), \quad (2.8)$$

which is independent of the black hole mass in the $\mu \gg \ell$ limit. In other words the near-horizon region of very large black holes has the same AdS scale curvature for all (very large) black hole masses. Note that this universal curvature is not equal to the curvature of empty AdS vacuum with the same cosmological constant but rather an $O(1)$ multiple of the vacuum value.

At the quantum level AdS black holes emit Hawking radiation [11] but due to the confining gravitational potential of the asymptotic AdS background the Hawking radiation cannot escape to infinity and is reflected back towards the black hole. The fate of an evaporating AdS-Schwarzschild black hole depends very much on which of the two branches it belongs to. A large black hole reabsorbs most of the reflected Hawking radiation and eventually settles into an equilibrium where the black hole continually exchanges radiation with a surrounding 'atmosphere'. In section 3 we obtain static semiclassical solutions that describe such equilibrium configurations. A small AdS black hole, on the other hand, radiates away its mass in a runaway process like an ordinary Schwarzschild black hole in asymptotically flat spacetime. The Hawking radiation is reflected back from infinity but it is not reabsorbed by the black hole at a high enough rate to keep up with the accelerating evaporation process. At the end of day the small black hole has completely evaporated and its energy is instead contained in a spherically symmetric 'star' made of self-gravitating radiation [9, 10].

2.2 Self-gravitating radiation

Classical solutions that describe a spherical concentration of self-gravitating radiation in a spacetime with negative cosmological constant are easily found [9, 10]. Let us briefly

review their construction in four spacetime dimensions.² For simplicity we assume that the Hawking radiation consists of massless particles and model its gravitational effect by a spherically symmetric perfect fluid energy-momentum tensor,

$$T_{ab} = \rho(r)u_a u_b + p(r)(g_{ab} + u_a u_b), \quad (2.9)$$

with a linear equation of state,

$$\rho(r) = 3p(r). \quad (2.10)$$

Here u_a is the 4-velocity of the radiation fluid. This energy-momentum tensor is inserted into the Einstein equations along with a spherically symmetric metric ansatz,

$$ds^2 = -\frac{1}{\ell^2} \left(\frac{\rho_\infty}{\rho(r)} \right)^{1/2} dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} + 1 - \frac{2m(r)}{r} \right)} + r^2 d\Omega^2. \quad (2.11)$$

The normalization constant ρ_∞ , which is determined by the asymptotic large r behavior of the energy density,

$$\rho(r) \approx \frac{\rho_\infty}{r^4}, \quad (2.12)$$

is needed in order for the asymptotically AdS metric to have the right characteristic length ℓ . The Einstein equations then reduce to a pair of ordinary differential equations. One of the equations determines the so-called mass function in terms of the energy density,

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r), \quad (2.13)$$

and the other is the Oppenheimer-Volkoff equation,

$$\frac{d\rho(r)}{dr} = -\frac{4\rho(r)}{r} \frac{\left(\frac{4}{3}\pi r^3 \rho(r) + m(r) + \ell^{-2} r^3 \right)}{(\ell^{-2} r^3 + r - 2m(r))}, \quad (2.14)$$

with the effect of the negative cosmological constant included through the terms involving the AdS length scale ℓ .

The first order differential equations (2.13) and (2.14) can easily be solved numerically and turn out to have a one-parameter family of solutions labeled by the central energy density $\rho(0)$. The solutions are non-singular and describe a spherically symmetric density profile that monotonically decreases away from $r = 0$, as shown in figure 1. The asymptotic behavior of the energy density for $r \gg \ell$ is of the form (2.12), with ρ_∞ determined by the value of $\rho(0)$. The mass function vanishes at the origin, $m(0) = 0$, and asymptotically it behaves as

$$m(\infty) - m(r) = O(r^{-1}), \quad (2.15)$$

where $m(\infty)$ denotes the total mass of the AdS 'star'.

Formally the central energy density has no upper bound but new physics comes into play when $\rho(0)$ becomes very large. In string theory, for example, the equation of state of the radiation fluid is modified to $p \approx 0$ at very high energy densities approaching the Hagedorn scale reflecting the tendency to form long strings that exert very little pressure compared to a gas of massless particles at the same energy density.

²The corresponding solutions in five-dimensional spacetime are discussed in detail in [10].

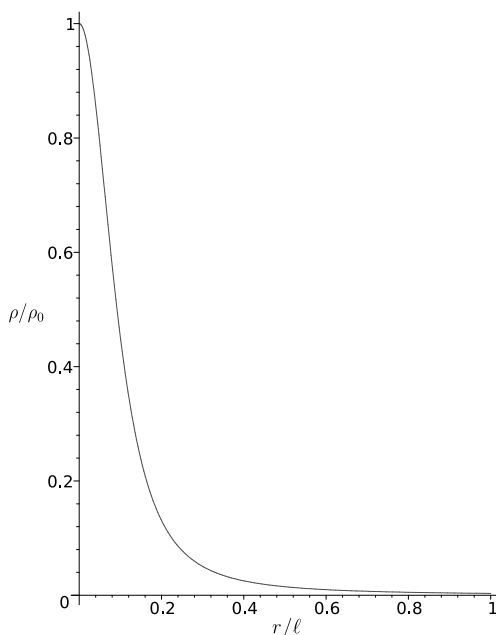


Figure 1: Density profile of spherically symmetric self-gravitating radiation.

The AdS star solutions have two interesting features that were pointed out in [10]. First of all $m(\infty)$ is bounded from above by a relatively small mass which is of order one in units of the AdS length ℓ . This means in particular that one of these self-gravitating radiation configurations can at most have a total mass that equals that of a small AdS-Schwarzschild black hole and never that of a large AdS-Schwarzschild black hole. This is one reason why we expect self-gravitating radiation to be the endpoint of evaporation of small black holes while large black holes must settle into equilibrium with a surrounding radiation atmosphere.

The second curious feature observed in [10] is that the total mass is not a monotonic function of the central energy density, which is the parameter that labels the 'star' solutions. Instead $m(\infty)$ reaches its maximum value for a finite value of $\rho(0)$ and then drops somewhat lower

as $\rho(0) \rightarrow \infty$, oscillating about a limiting value. It was speculated in [10] that this corresponds to the onset of a radial instability as further discussed in [12, 13].

The conclusion that $m(\infty)$ is bounded from above is unchanged when we consider a gas made of strings rather than a gas of massless particles. In fact, the modified equation of state of the string gas is expected to precipitate the transition to a black hole at a somewhat *lower* total energy. This is because the $p \approx 0$ equation of state of the long string phase is too soft to support a non-singular AdS star once the energy density in the center reaches the string scale.

3. Semiclassical geometry

In this section we consider quantum corrections to adS black hole geometries. We look for static semiclassical solutions that describe spherically symmetric adS black holes in equilibrium with a surrounding atmosphere consisting of matter radiation. We use spherical reduction to make the semiclassical calculation tractable. At the end of the day we find that the energy density in radiation surrounding a very large black hole is small which suggests that the back-reaction problem is indeed dominated by s-wave modes and that the spherical approximation is justified.³

3.1 One-loop action

We start from the 3+1 dimensional Einstein-Hilbert action with a negative cosmological

³The reliability of the s-wave approximation to four-dimensional gravity is discussed in [14].

constant,

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g^{(4)}} \left(R^{(4)} + \frac{6}{\ell^2} \right), \quad (3.1)$$

and use a metric ansatz,

$$ds^2 = g_{ij}(x^0, x^1) dx^i dx^j + r(x^0, x^1)^2 d\Omega_2^2, \quad (3.2)$$

to perform a spherical reduction to the 1+1 dimensional classical action

$$S_0 = \frac{1}{4G} \int d^2x \sqrt{-g} \left(r^2 R - 4r \nabla^2 r - 2(\nabla r)^2 + 2 + 6r^2 \ell^{-2} \right). \quad (3.3)$$

We also include the spherically reduced action of N scalar fields that are minimally coupled in 3+1 dimensions,

$$S_m = -2\pi \int d^2x \sqrt{-g} r^2 \sum_{i=1}^N (\nabla f_i)^2. \quad (3.4)$$

For $N \gg 1$ the one-loop effective action describing the semiclassical back-reaction due to this matter sector is given by adding the following conformal anomaly term to the action [15–25],

$$S_q = -\frac{N}{12} \int d^2x \sqrt{-g} \left(\frac{1}{2} R \square^{-1} R - \frac{6(\nabla r)^2}{r^2} \square^{-1} R - 6R \ln r \right). \quad (3.5)$$

We can express the back-reaction term in local form using auxiliary fields ψ and χ [25]:

$$S_q = -\frac{N}{12} \int d^2x \sqrt{-g} \left(R(\psi - 6\chi - 6 \ln r) + \frac{1}{2} (\nabla \psi)^2 - 6 \nabla \psi \nabla \chi - 6\psi \frac{(\nabla r)^2}{r^2} \right) \quad (3.6)$$

Quantum corrections due to this term are small when $NG/\ell^2 \ll 1$. This can be arranged for any given values of $N \gg 1$ and G by choosing a sufficiently large AdS length ℓ .

3.2 Static solutions

We look for static solutions of the semiclassical equations of motion in conformal gauge, $g_{\mu\nu} = f(x)\eta_{\mu\nu}$ with the classical matter fields f_i set to zero. The remaining fields only depend on the coordinate x and the semiclassical equations reduce to

$$\psi'' + (\ln f)'' = 0, \quad (3.7)$$

$$\chi'' - \frac{(r')^2}{r^2} = 0, \quad (3.8)$$

$$r^2 \left(\frac{1}{2} (\ln f)'' - \frac{3f}{\ell^2} + \frac{r''}{r} \right) = -NG \left(\left(\psi \frac{r'}{r} \right)' + \frac{1}{2} (\ln f)'' \right), \quad (3.9)$$

$$(r')^2 + rr'' - f \left(1 + \frac{3r^2}{\ell^2} \right) = -NG \left(\frac{1}{6} (\ln f)'' + \frac{r''}{r} \right), \quad (3.10)$$

where a prime denotes a derivative with respect to x . The constraint equation corresponding to this gauge choice is

$$-rr'' + rr'(\ln f)' = NG \left(-\frac{1}{6} \psi'' + \frac{r''}{r} + (\ln f)' \left(\frac{1}{6} \psi' - \chi' - \frac{r'}{r} \right) + \frac{1}{12} (\psi')^2 - (\psi \chi')' \right). \quad (3.11)$$

Our aim is to find the leading semiclassical correction to the mass of a black hole. For this we need an approximate solution of these equations that is first order in NG/ℓ^2 and valid for large r . Since there is no explicit dependence on coordinate x in the system (3.7)–(3.11), we look for a solution of the fields ψ , χ' , f and r' as a function of the field $r(x)$.

First of all it is straightforward to verify that the AdS-Schwarzschild spacetime,

$$f(r) = f_0(r) = \frac{r^2}{\ell^2} + 1 - \frac{\mu}{r}, \quad (3.12)$$

$$r'(r) \equiv \frac{dr}{dx} = f_0(r), \quad (3.13)$$

is recovered as a solution of equations (3.9)–(3.11) when $N = 0$.

Next we consider the back-reaction due to the one-loop correction term using NG/ℓ^2 as a small expansion parameter. The first step is to determine the $\mathcal{O}(N^0)$ behavior of the auxilliary fields ψ and χ . Inserting the classical solution (3.12) for f and $\frac{dr}{dx}$ into equations (3.7) and (3.8) leads to

$$\frac{d}{dr} \left(f_0 \frac{d\psi}{dr} + \frac{df_0}{dr} \right) = 0, \quad (3.14)$$

$$\frac{d}{dr} \chi' = \frac{f_0}{r^2}. \quad (3.15)$$

We do not need the explicit form of $\chi(r)$. Only derivatives of χ , and not the field itself, appear in the equations of motion and for our purposes it is enough to solve for χ' .

Equation (3.15) is easily integrated,

$$\chi' = \frac{r}{\ell^2} + \chi_1 - \frac{1}{r} + \frac{\mu}{2r^2}. \quad (3.16)$$

To fix the constant of integration χ_1 we require that χ and its derivatives with respect to r be free of singularities at the horizon. From the relation

$$\chi' \equiv \frac{d\chi}{dx} = f_0 \frac{d\chi}{dr}, \quad (3.17)$$

which holds at the level of approximation that we are working with, we immediately see that χ' must vanish at the horizon $r = r_s$, i.e. that

$$\chi_1 = \frac{1}{2} \left(\frac{1}{r_s} - \frac{3r_s}{\ell^2} \right). \quad (3.18)$$

This expression diverges in the $r_s \rightarrow 0$ limit but this is not a problem because the semiclassical equations that we are working with are only valid for $r^2 \gg NG$ and this puts a lower limit on the size of semiclassical black holes that they apply to.

A first integral of equation (3.14) gives

$$f_0 \frac{d\psi}{dr} + \frac{df_0}{dr} = \psi_1. \quad (3.19)$$

The integration constant is fixed by the requirement that $d\psi/dr$ be non-singular at the horizon,

$$\psi_1 = \frac{1}{r_s} + \frac{3r_s}{\ell^2}. \quad (3.20)$$

A second integration of equation (3.14) then gives

$$\psi(r) = -\ln f_0(r) + \psi_1 \int^r \frac{d\tilde{r}}{f_0(\tilde{r})}. \quad (3.21)$$

The asymptotic expansion of $\psi(r)$ at large r is given by

$$\psi(r) = -2 \ln r + \psi_0 - \frac{\ell^2 \psi_1}{r} + \mathcal{O}(r^{-2}), \quad (3.22)$$

where ψ_0 is another integration constant, which will not affect any of our conclusions and can be left undetermined.

Next, we use the results (3.16) and (3.22) to solve the constraint equation (3.11) to order $\mathcal{O}(NG/\ell^2)$. Writing a first-order ansatz for the fields $f(r)$ and $\frac{dr}{dx}$,

$$f(r) = f_0(r) + \frac{NG}{\ell^2} D(r), \quad \frac{dr}{dx} = f_0(r) + \frac{NG}{\ell^2} C(r), \quad (3.23)$$

and expanding to order $\mathcal{O}(NG/\ell^2)$, the constraint equation (3.11) can be written

$$\begin{aligned} \frac{d}{dr} \left(\frac{D(r) - C(r)}{f_0} \right) &= \frac{\ell^2}{r f_0} \left(\frac{d^2 f_0}{dr^2} + \frac{6}{r} \frac{df_0}{dr} + \left(\frac{1}{f_0} \frac{df_0}{dr} \right) \left(\psi_1 - \frac{df_0}{dr} - 6\chi' - \frac{6}{r} f_0 \right) \right. \\ &\quad \left. + \frac{1}{2f_0} \left(\psi_1 - \frac{df_0}{dr} \right)^2 - 6 \frac{d}{dr} (\psi \chi') \right). \end{aligned} \quad (3.24)$$

Integrating this equation gives

$$C(r) - D(r) = \ln r + \frac{1}{2} - \frac{1}{2} \psi_0 + \mathcal{O}(r^{-2} \ln r). \quad (3.25)$$

There is an integration constant which we have set to zero to preserve the asymptotic value of the cosmological constant.

Expanding equation (3.10) to order $\mathcal{O}(NG/\ell^2)$, we obtain another linear equation involving $C(r)$ and $D(r)$,

$$r \frac{dC}{dr} + \left(2 + \frac{r}{f_0} \frac{df_0}{dr} \right) C - \left(1 + \frac{3r^2}{\ell^2} \right) D = -\frac{\ell^2}{6} \left(\frac{d^2 f_0}{dr^2} + \frac{6}{r} \frac{df_0}{dr} \right). \quad (3.26)$$

By using the result (3.25), we can finally solve for the asymptotic expansions of $C(r)$ and $D(r)$,

$$C(r) = -3 \ln r - \frac{5}{6} + \frac{3}{2} \psi_0 + \frac{c_0}{r} + \mathcal{O}(r^{-2} \ln r) \quad (3.27)$$

$$D(r) = -4 \ln r - \frac{4}{3} + 2\psi_0 + \frac{c_0}{r} + \mathcal{O}(r^{-2} \ln r) \quad (3.28)$$

where c_0 is an integration constant with dimensions of length. We find it convenient to express it in terms of a characteristic length scale of the black hole geometry,

$$c_0 = \alpha r_s, \quad (3.29)$$

with α an undetermined dimensionless constant. It is straightforward to show that with these asymptotic expansions for $C(r)$ and $D(r)$ the remaining equation of motion (3.9) is also satisfied to $\mathcal{O}(NG/\ell^2)$, which provides a check on the solution that we have found.

4. Energy considerations

In this section we estimate the leading order quantum corrections to the total energy of a large AdS-Schwarzschild black hole and based on this we can estimate the energy density of the radiation atmosphere that surrounds the black hole. We define the total energy in terms of a Brown-York quasilocal stress tensor [26] adapting the subtraction procedure of Balasubramanian and Kraus [27] to our spherically reduced problem.⁴ We first consider the classical 1+1 dimensional theory and confirm that the formalism gives the correct mass for a classical adS-Schwarzschild black hole. We then include $\mathcal{O}(NG/\ell^2)$ corrections and obtain an order of magnitude estimate of the total energy residing in the semiclassical black hole atmosphere.

4.1 Classical total energy

The 1+1 dimensional classical action (3.3) needs to be supplemented by a boundary term in order to have a well posed variational problem for metric variations $\delta g_{\mu\nu}$ that vanish at the $r \rightarrow \infty$ boundary. This can ultimately be traced to the $r^2 R$ and $r \nabla^2 r$ terms in (3.3), both of which contain second derivatives acting on field variables. The boundary term whose variation cancels the unwanted terms from the variation of the bulk classical action can be written

$$S_b = \frac{1}{2G} \int dt \sqrt{-\gamma} (r^2 \gamma^{tt} \nabla_t n_t + n^\mu \nabla_\mu (r^2)). \quad (4.1)$$

Here we are using the two-dimensional coordinate time t as the parameter along the boundary and the one-dimensional boundary metric γ_{tt} is induced from the two-dimensional $g_{\mu\nu}$. The covariant derivatives in this expression are with respect to the two-dimensional metric and n^μ are the components of the unit normal to the boundary. The combination $\gamma^{tt} \nabla_t n_t$ is the one-dimensional analog of the trace of the extrinsic curvature of the boundary while the $n^\mu \nabla_\mu (r^2)$ term can be traced to the spatial curvature of the two-spheres that we are reducing on. The boundary term (4.1) can be written in a more compact way,

$$S_b = \frac{1}{2G} \int dt \sqrt{-\gamma} \nabla_\mu (r^2 n^\mu), \quad (4.2)$$

where we have used that the tensor $\nabla_\mu n_\nu$ is tangential to the boundary.

In conformal gauge, $g_{\mu\nu} = f(x) \eta_{\mu\nu}$, we find the following "bare" quasilocal stress tensor

$$T^{tt} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S}{\delta \gamma_{tt}} = \frac{1}{2G} \gamma^{tt} n^a \nabla_a (r^2) = -\frac{1}{G} \frac{r}{\sqrt{f_0(r)}}, \quad (4.3)$$

where we have used the classical value (3.13) for the derivative $\frac{dr}{dx}$. This leads to a bare total energy,

$$\varepsilon = -\frac{r f_0(r)}{G} = \frac{1}{G} \left(-\frac{r^3}{\ell^2} - r + \mu \right), \quad (4.4)$$

⁴See [28, 29] for alternative approaches to regulating conserved quantities in asymptotically AdS space-times.

that diverges in the $r \rightarrow \infty$ limit. Following [27] we introduce a boundary counterterm in the classical action to cancel this divergence. In one dimension such a counterterm can only be a function of the induced metric γ_{tt} and the scalar field r [30] and we find that

$$S_{ct} = -\frac{1}{G\ell} \int dt \sqrt{-\gamma} \left(r^2 + \frac{\ell^2}{2} \right), \quad (4.5)$$

achieves the desired cancellation. With the addition of this counterterm we obtain a finite total energy,

$$\varepsilon = \lim_{r \rightarrow \infty} \left(-\frac{r f_0(r)}{G} + \frac{\sqrt{f_0(r)}}{G\ell} \left(r^2 + \frac{\ell^2}{2} \right) \right) = \frac{\mu}{2G}, \quad (4.6)$$

which equals the classical black hole mass.

4.2 Semi-classical energy

We now want to include leading order semiclassical corrections in these expressions in order to estimate the contribution of the black hole atmosphere to the total energy. In the classical boundary term (4.2) the function that accompanies the normal vector n^μ inside the parenthesis is the function that multiplies the Ricci scalar in the bulk classical action (3.3). The natural semi-classical generalization of the boundary term is therefore

$$S'_b = \frac{1}{2G} \int dt \sqrt{-\gamma} \nabla_\mu (h n^\mu), \quad (4.7)$$

where $h = r^2 - \frac{NG}{3}(\psi - 6\chi - 6 \log r)$ is the function that multiplies R in the one-loop corrected bulk action $S_0 + S_q$ in (3.3) and (3.6).

The semiclassical quasilocal stress tensor can now be computed and one finds new divergences in the bare total energy. These divergences are canceled if we include a one-loop correction in the boundary counterterm,

$$S'_{ct} = -\frac{\ell}{G} \int dt \sqrt{-\gamma} \left(\frac{r^2}{\ell^2} + \frac{1}{2} - \frac{NG}{2\ell^2} \left(\psi + \frac{13}{3} \right) \right). \quad (4.8)$$

The remaining finite total energy is given by

$$\varepsilon = \frac{1}{2G} \left(\mu + \frac{NG}{\ell^2} \left((1 - \alpha) r_s - \frac{5\ell^2}{3r_s} \right) \right), \quad (4.9)$$

where α comes from the undetermined integration constant in (3.29). For large black holes, with $r_s \gg \ell$, this reduces to

$$\varepsilon \approx \frac{1}{2G} \left(\mu + (1 - \alpha) \left(\frac{NG}{\ell^2} \right) r_s \right). \quad (4.10)$$

The $\mathcal{O}(\frac{NG}{\ell^2})$ term in the total energy (4.10) provides an order-of-magnitude estimate of the energy carried by the radiation bath surrounding the black hole. This energy increases linearly with r_s while the area of the black hole is proportional to r_s^2 . It follows that the

average proper energy density in the radiation outside the black hole must go to zero in the $r_s \rightarrow \infty$ limit.⁵

5. Discussion

Our semiclassical calculation supports the notion that, despite having a very high Hawking temperature, a large AdS black hole is a cold object in the sense that observers in free fall outside the black hole do not see high-temperature thermal radiation. The absence of high-energy thermal radiation can also be seen via a more direct calculation [31] that utilizes the global embedding of the AdS-Schwarzschild geometry into a higher dimensional flat spacetime along the lines of [32].

Our results have implications for string thermodynamics with an AdS infrared regulator [6]. At low total energy the stable configuration of such a string gas is an AdS star in local thermal equilibrium. Now consider increasing the total energy in order to study ever higher energy densities at the center of the AdS star. This can only be carried so far because once the energy density in the center becomes of order the string scale the system is unstable to gravitational collapse. This occurs at a total energy of order the AdS scale and for higher total energies the stable configuration is a large AdS black hole. The Hawking temperature of such a black hole can be arbitrarily high if the black hole is large enough but from the point of view of observers in free fall it seems that we are not probing high temperatures at all. The dual gauge theory does provide a thermal description in terms of a high-temperature plasma of deconfined gluons at the Hawking temperature but this is a very indirect and non-local description of the gravitational physics.

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⁵It is of course possible that the leading behavior of the $\mathcal{O}(\frac{NG}{\ell^2})$ term in the total energy should be interpreted as coming from a semiclassical correction to the mass of the black hole itself rather than as energy carried by surrounding radiation but in this case the energy density of the radiation bath around a large AdS black hole is even smaller than our (already very small) estimate.

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